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APPLICATION OF CREDIT RATING MODELS IN FINANCIAL SECTOR

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Abstract:
This paper presents some applications of credit rating models that exploit the probability of default and loss in case of default when calculating the risk premium. The paper outlines the general risk-based pricing framework and illustrates it through numerical estimation of risk premium for reverse factoring, a banking product that is widely used in international trade. Based on the financial reports, we have calculated the probabilities of default for Serbian non-financial enterprises, and the corresponding risk premium. The results obtained have enabled assessing the product's potential on the local market.

Key words:
risk premium,
risk-based pricing,
reverse factoring.

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1. INTRODUCTION

Credit rating models essentially map borrowers or exposures into categories based on quantitative credit risk measures. These measures usually include probability of default (PD), or a combination of PD and loss given default (LGD). The general framework presented herein is suitable for incorporation of any set of explanatory variables. It is also applicable to different types of products and basically all kinds of credit exposures: corporate, small and medium -sized enterprises, retail, financial institutions, or sovereigns.

Desirable properties of any good credit rating model are accuracy, simplicity, feasibility, transparency, and clear economic interpretation. Accuracy refers to a model's ability to provide an adequate statistical fit to the observed data. Simplicity means that accuracy should not be achieved at the expense of overparameterization, which happens if one uses way too many explanatory factors in the model. Feasibility pertains to the choice of explanatory variables that can be retrieved from data that are actually available, but also to the principle that the model should not rely on the techniques that are too computationally intensive. Transparency of the model is related to the way it is documented methodologically and technically – anyone within the company that implements the model in everyday business should be familiar with each step. Finally, clear economic interpretation refers to the choice of parameters that are not merely statistically significant, but also have a proper economic intuition behind them. The last property is crucial for the ability of experts to judge the soundness of predictions obtained from the model. This is the weakest point of many credit scores based on data mining, a practice that is becoming quite popular nowadays.

2. BUILDING A CREDIT RATING MODEL

Having in mind the desirable properties of the credit rating model discussed in the Introduction, let us establish the key steps in building a model based on PD. A more detailed procedure can be found in Fernandes (2005) or Hayden and Porath (2011). The data for the explained variable usually come in the form of default indicators for each borrower or exposure. The indicator has a value of 1 in case of default, and 0 otherwise. There are various interpretations of "default". For instance, we can adopt the accounting view, in which the borrower defaults if viewed to have a small chance of further payments. Banks often use the trigger of 90 days past due prescribed by the Basel regulatory framework. Alternatively, the model can work with the legal status (liquidation/restructuring) in case of companies, behavioral parameters (*e.g.* data on unpaid utility bills or taxes), or standard ISDA credit event triggers[1] in case of sovereigns, financial institutions or large corporate exposures.

Explanatory variables may contain all available data that are potentially useful for creating risk factors, such as borrower-specific factors (credit history, current level of liabilities, financial ratios), other specific factors in case of firms (industry sector, size, type of business), categorical variables for individuals (age group, gender, relationship status, ZIP code), or economy-wide factors (GDP growth, exchange rates, interest rates and other macroeconomic variables). Guyon and Elisseeff (2003) provide a good introduction to the variable selection process.

In order to give an example of a factor model for PD, we will use a pooled panel consisting of credit exposures (in the form of loans, credit lines, bonds, *etc.*), observed across

1 See www.isda.org.

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several years. We shall start by defining the *creditworthiness index* (or the payment ability) as:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_K x_{Ki} + \varepsilon_i = \mathbf{x}_i' \boldsymbol{\beta} + \varepsilon_i, \qquad (1)$$

where
$i=1,2, \ldots, n$ labels exposure-year,
$\mathbf{x}_i = \{x_{ki}\}_{k=1,\,2,\,\ldots,\,K}$ are repayment ability factors,
$\boldsymbol{\beta} = \{\beta_k\}_{k=1,\,2,\,\ldots,\,K}$ are the (initially unknown) model coefficients, and ε_i are error terms with mean zero. Note that y_i is *unobservable*, *i.e.* it represents a latent variable in the model. Its relationship with observable data is that $y_i \geq 0$ if there was no default of exposure-year i ($d_i = 0$), and $y_i < 0$ if there was a default of exposure-year i ($d_i = 1$). Then, conditional probability of default of exposure-year i is:

$$p_i = \mathbb{P}(d_i = 1) = \mathbb{P}(y_i < 0) = F(-\mathbf{x}_i' \boldsymbol{\beta}), \qquad (2)$$

where $F(\cdot)$ denotes the distribution function. Usual assumptions for $F(\cdot)$ are either normal or logistic distribution. In the former case, the model corresponds to probit, while in the latter to logit. The model parameters can be estimated via maximum likelihood (ML). If the sample is truly random, then d_i follows a Bernoulli distribution, and the likelihood function is given by

$$L(\boldsymbol{\beta}) = \prod_{i=1}^{n} (p_i)^{d_i} (1 - p_i)^{1-d_i}. \qquad (3)$$

We can estimate $\boldsymbol{\beta}$ by maximizing

$$\ln L(\boldsymbol{\beta}) = \sum_{i=1}^{n} [d_i \ln F(-\mathbf{x}_i' \boldsymbol{\beta}) + (1 - d_i) \ln F(\mathbf{x}_i' \boldsymbol{\beta})]. \qquad (4)$$

For instance, Hayden (2011) illustrates such an estimation for corporate exposures. The standard likelihood ratio or other asymptotically equivalent tests can be used to assess statistical significance of the chosen set of factors x_i. If $\hat{\beta}$ is the ML estimate of the vector of model coefficients, the forecast of the conditional PD for exposure-year i is then simply:

$$\hat{p}_i = F\left(-\mathbf{x}_i' \widehat{\boldsymbol{\beta}}\right). \qquad (5)$$

Given any exposure-year, either in or out of the calibration sample, we can forecast its conditional PD directly from the observable risk factors x_i.

The model estimation is usually accompanied by several econometric issues. For instance, *multicollinearity* introduces inconsistency in estimation of parameters and assessment of their significance. It is commonly solved by excluding collinear factors, but alternative approach is to use the principal component analysis to achieve orthogonality between the factors and (if necessary) reduction of dimensionality. *Linear separability* appears whenever some factors may have cut-off points that appear to explain 100% of defaults. This problem can be solved again by excluding such factors, or through a technique called penalized likelihood (see Heinze *et al.*, 2002). The common issue of *missing data* can be resolved by "clever" interpolation, if excluding factors reduce the quality of fit significantly. Sometimes, the panel is relatively small along both dimensions, and thus not representative (*e.g.* a credit portfolio of a small bank observed over a couple of non-recession years). This leads to *sparsity*, usually in the form of having too few default indicators with a value of 1. In that case, β coefficients may become biased, as discussed by Pluto *et al.* (2011). This bias can be removed by using the penalized likelihood. On the other hand, big data often contain a lot of *outliers*, to which the ML estimation in the logistic regression is highly sensitive. A possible solution is to remove the outliers via Winsorizing. Some examples can be found in Sarkar *et al.* (2011).

The end-result of a credit rating model is mapping of PDs into rating categories. To show how this can be done, note that PD takes real values between 0 and 1. To create alphanumerical ratings, this unit interval needs to be divided into k discrete categories. Desirable properties of this division are stability and discriminatory power, as we would like to have as few transitions between non-adjacent categories as possible and default rate should increase monotonously with category. A good way to achieve this is through the method of k-means clustering, which determines the optimal number of rating categories by maximizing

$$\frac{BSS/(k-1)}{WSS/(n-k)}, \qquad (6)$$

where k is the number of clusters, n is the number of observations, BSS is the sum of squared deviations between the clusters and the entire sample, while WSS is the sum of squared deviations within each cluster.[2]

Any credit rating model needs to be validated. Validation uses out-of-the-sample observations to determine the predictive ability of the rating model. It usually consists of two phases:

(1) validation of the model's *discriminatory power*, *i.e.* the ability to predict defaults as accurately as possible; and
(2) validation of the *goodness of fit*, *i.e.* the ability to forecast the actual default rate for each rating category. Most methods applied in validation of discriminatory power are based on comparison of distributions between defaulted and non-defaulted exposure-years.

The examples of such methods include Cumulative Accuracy Profile, Accuracy Ratio, Receiver Operating Characteristic (ROC), Area Under ROC Curve, Brier score, Conditional entropy, Kendall's τ, Somers' D, *etc.* Validation of goodness of fit applies usual statistical methods of comparison between the candidate model and the observed distributions. It includes statistical tests such as Kolmogorov-Smirnov and Hosmer-Lemeshow (see Hosmer *et al.*, 2013). A detailed overview of validation techniques can be found, for example, in BIS (2005) or OeNB (2004).

The final step in development of a credit rating model involves the choice among the available alternatives, since several competing models may pass the validation. Typically, Akaike or Bayesian information criterion are used to rank these models, awarding explanatory power and penalizing excessive parameterization.

2 See, for example, Siddiqi (2005).

3. RISK-BASED PRICING

In commercial banking, credit rating models are mostly used in the process of loan approval. However, they have many other useful applications, such as risk-based pricing and assessment of risk capital.[3] Technically, these two concepts are closely related to expected and unexpected loss. Risk-based pricing is the process in which the price of a credit product, such as loan, is made sensitive to the credit quality of the borrower. The alternative of a common (*e.g.* median) interest rate charged to all the buyers of the same product leads to a portfolio that *may* have appropriate expected loss only on aggregate. In addition, this practice of binary loan approval with a single interest rate for each product leads to a pooling equilibrium with adverse selection of borrowers. The lack of screening attracts bad credit through comparably lower interest rates, and disincentivizes good credit through comparably higher interest rates. Risk-based pricing allows the lender to differentiate between good and bad credit and charge a different (essentially continuous) risk premium based on the borrower's credit quality. Here, we describe a simple model that illustrates how the premium can be calculated.

Consider a loan with an outstanding principal *N*. If the loan is structured as an annuity paid over *T* periods in constant amounts a, in the absence of credit risk we will have:

$$N = \sum_{t=1}^{T} \frac{a(r,\, N)}{(1+r)^t}. \quad (7)$$

The previous equality gives the payment amount $a(r,N)$, which depends on the (risk-free) interest rate *r* and the principal *N*. However, in the presence of credit risk, the equality does not hold, since the borrower may default on the payments before period *T*. To capture this possibility, we will assume that in each period the borrower's conditional probability of default is *PD*. Conditionally on default, the lender recovers the fraction equal to the loan's recovery rate, *RR=1-LGD*. The payments then satisfy the following equation:

$$N = \sum_{t=1}^{T} \frac{(1-PD)^{t-1} \cdot PD \cdot RR \cdot EAD + (1-PD)^t \cdot a(r^*,\, N)}{(1+r)^t}, \quad (8)$$

where *EAD* stands for exposure at default, while r^* is the risk-adjusted interest rate. Essentially, it is the risk-free rate *r* plus the risk premium. It may also incorporate any accompanying fees that the lender charges. In this case, the payment amount cannot be obtained in a closed form, unlike the standard annuity formula. Thus, we have to solve equation (8) numerically for r^*, using other values as given. In particular, *PD* (or both *PD* and *LGD*) should be provided from a rating model.

It is worth mentioning that the risk-adjusted interest rate r^* should be determined self-consistently in equilibrium, since it may have an ex-post influence on credit risk factors. To see this, suppose that we apply a rating model for corporate borrowers and it turns out that some measure of leverage is a significant determinant of default. If the lender approves the loan, the borrower's leverage will immediately increase as a result. This, in turn, changes the PD, which becomes a function of the interest rate and the outstanding loan amount:

$$PD_t = PD_t(a(r^*, N)) = PD_t(r^*, N). \quad (9)$$

Hence, PD becomes an implicit function of r^*, and this should be taken into account. In sum, the interest rate that incorporates credit risk premium depends on PD, LGD, amortization scheme and outstanding loan amount.

4. REVERSE FACTORING PREMIA

As an illustration of possible applications of credit rating models, in this section we demonstrate the calculation of risk premia for hypothetical reverse factoring (RF) contracts. Reverse factoring (also known as supply-chain financing) is a type of product that is becoming increasingly popular in international trade (see BIS, 2014). Essentially, in RF the bank or a factoring company (called the "factor") interposes between its client (usually an importer) and their suppliers (exporters), by making the immediate payment for goods or services to a supplier in exchange for future installments from the client. The bank benefits from the informational asymmetry it possesses and the trade discounts offered by the supplier for early payments. The product is different than the common letter of credit, which can delay new business due to administrative burdens and costs. The letter of credit that bank opens for the importer in favor of the supplier blocks the importer's credit line and requires much documentation specifying the imported goods and payment terms. Only after the complete receipt of documents will the payment to the supplier be initiated. RF is a reversed process of classic factoring, in which the first contact of the factoring company is the importer instead of the supplier.

For the calculation of the hypothetical RF premia, we use publicly available data on all non-financial enterprises in Serbia that had official financial statements at the end of 2012 and were registered with Serbian Business Registers Agency at that time (a total of 78,011 companies). We apply a logit-based rating model as outlined in Section 2. The model has been trained on a sample of 21,682 firm-years, with a total of 623,598 firm-year observations. All companies have been segregated into their respective industry sectors according to their official NACE segmentation.

Table 1 summarizes the results. The rating model has found 17 rating categories through the *k*-means procedure (see Section 2). The expected default rates are shown in the second column of Table 1 for each category. As a crude guideline, category 1 has an expected default rate that corresponds to AAA–A rating of Standard & Poor's (2014) global one-year corporate default rates between 1981–2014. Similarly, categories 2–5 correspond to BBB, categories 6–10 to BB, categories 11–14 to B, while categories 15–17 would roughly fall into CCC–D rating of S&P.

The RF premia are summarized in the third column of Table 1 following the algorithm described in Section 3. We assume that the bank charges a fixed fee of 2%, that the 3-month Euribor rate of –0.038% can be used as a proxy for the risk-free rate,[4] and that the RF products have maturity of

3 See, for example, Glanz and Moon (2008).

4 Data for September 16, 2015.

3 months each. LGD is not modeled explicitly here. Instead, we assume a conservative value of 100%.

Table 1 - Summary of credit rating categories

Rating Category	Expected Default Rate	Reverse Factoring Premium
1	0.1172%	0.1196%
2	0.1980%	0.2023%
3	0.3078%	0.3148%
4	0.4453%	0.4561%
5	0.5964%	0.6117%
6	0.7682%	0.7893%
7	0.9811%	1.0103%
8	1.2908%	1.3333%
9	1.7259%	1.7907%
10	2.3105%	2.4116%
11	3.1290%	3.2934%
12	4.5395%	4.8487%
13	7.4469%	8.2039%
14	13.4403%	15.8313%
15	27.0547%	37.8167%
16	43.3295%	77.9598%
17	62.8665%	172.6203%

Table 2 - Market sizing results across sectors

NACE codes	Market share	Payables per sector
A	4.50%	4.02%
B	4.95%	5.39%
C	20.96%	23.29%
D	14.74%	9.63%
E	2.20%	1.64%
F	11.94%	11.42%
G	17.52%	25.02%
H	6.98%	5.35%
I	0.00%	0.00%
J	3.91%	3.56%
K	5.21%	3.88%
L	0.01%	0.01%
M	4.36%	4.47%
N	0.59%	0.73%
O	0.00%	0.01%
P	0.03%	0.03%
Q	0.05%	0.06%
R	1.96%	1.37%
S	0.09%	0.13%

To assess the potential for RF market in Serbia, we track the accounts payable across sectors and rating categories. Table 2 contains the breakdown of market share and accounts payable for each industry sector.[5] The following four sectors dominate the Serbian market, with a 65% of total market share:

C – Manufacturing
D – Electricity, gas, steam and air conditioning supply
F – Construction
G – Wholesale and retail trade.

Figs. 1–5 illustrate the distribution of total accounts payable of the entire market and its major sectors during 2012 for each rating category, as well as the corresponding number of companies. The distributions are presented for all the rating categories, except for the worst three, which have the annual probability of default (PD) above 20%.

Distribution of accounts payable across the rating categories for the entire market is shown in Fig. 1 (dark bars). To some extent, it follows the distribution of the number of companies (light line). The exceptions are intermediate rating categories (7–12), which are just below the investment grade, with an RF premium between 1.0 and 4.8%.

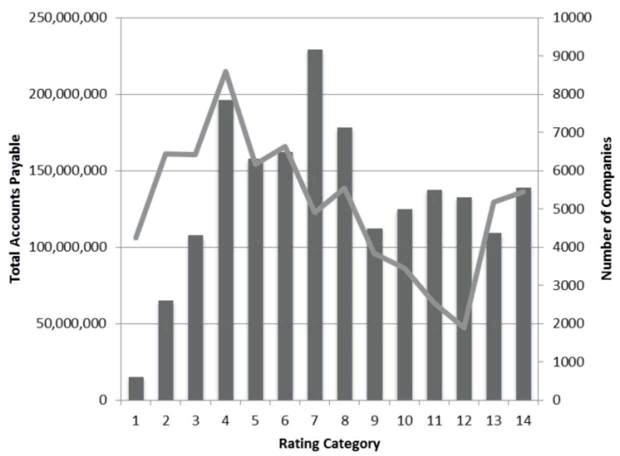

Fig. 1: Accounts payable for the entire market Left axis: accounts payable Right axis: number of companies

Distribution of accounts payable for the four key sectors of the Serbian economy are shown in Figs. 2–5. They have a relatively high correlation with the distribution of companies per rating category. The highest potential for low-risk RF activities lies with the companies having rating category below grade 9. As the PD increases, so does the risk of RF. However, the RF premium captures the associated risk, thereby increasing the potential profitability.

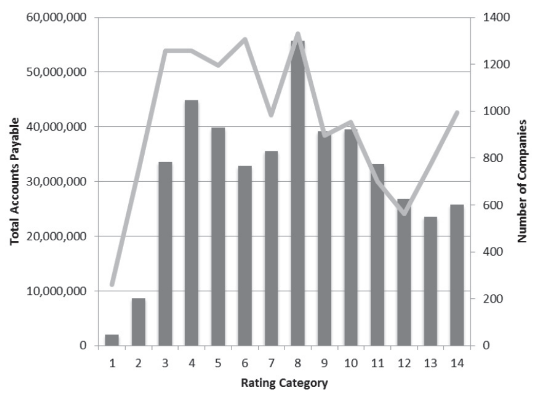

Fig. 2: Accounts payable, manufact. firms Left axis: accounts payable Right axis: number of companies

5 Breakdown follows the NACE classification of industries, http://ec.europa.eu/competition/mergers/cases/index/nace_all.html

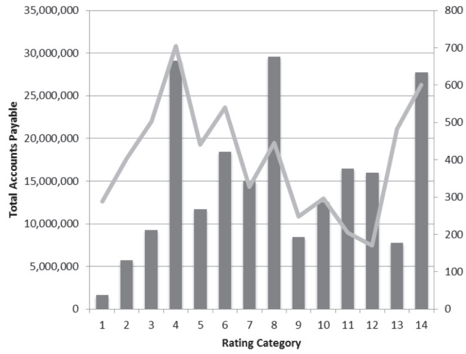

Fig. 3: Accounts payable, energy firms Left axis: accounts payable Right axis: number of companies

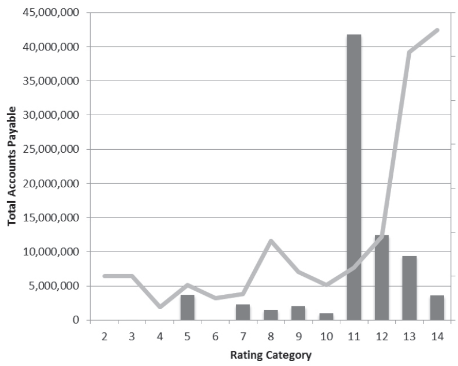

Fig. 4: Accounts payable, construction firms Left axis: accounts payable Right axis: number of companies

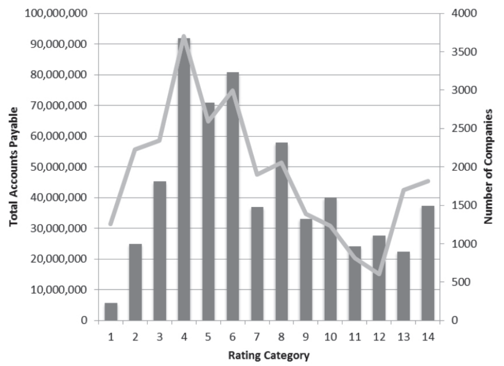

Fig. 5: Accounts payable, W&R trading firms Left axis: accounts payable Right axis: number of companies

Fig. 4 represents the RF potential of the construction sector, having a distribution that is skewed towards higher credit risk. This sector has suffered the heaviest financial losses during and after the global financial crisis. The current NPL ratio of this sector is around 54%.

5. SUMMARY

The aim of this paper was to illustrate the wide potential of credit rating models that goes beyond the simple loan approval process. We provide an overview of a general framework for development and validation of a linear factor-based rating model. One of the main outputs of the model is probability of default, which can be used as an input in calculation of risk premia. Risk premia differentiate borrowers by their ex-ante creditworthiness, thus preventing the adverse selection problem. The paper also derives the general equation that can be used in numerical calculation of credit risk premium for each individual loan.

As an illustration, we show the results for reverse factoring premia. The results are obtained numerically using a database of Serbian non-financial enterprises. They indicate that the market potential of reverse factoring is quite high, especially in intermediate rating categories. Given that the Serbian economy faces significant structural imbalances, trade financing may serve as a tool that could remove some of the inefficiencies resulting from informational asymmetries between foreign suppliers and local importers. However, the underlying assumption is that the local banks will use an adequate credit risk assessment in the process.

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PRIMENE MODELA KREDITNIH REJTINGA U FINANSIJSKOM POSLOVANJU

Apstrakt:

Ovaj rad prikazuje neke od primena modela kreditnih rejtinga koje se oslanjaju na verovatnoću neizmirenja i gubitak u slučaju neizmirenja pri računanju premija za rizik. U radu je dat opšti okvir za procenu kamatne stope koja uključuje premiju za rizik, na primeru numeričke procene premije za rizik u obrnutom faktoringu, bankarskom proizvodu koji ima široku primenu u međunarodnoj trgovini. Na osnovu finansijskih izveštaja izračunate su verovatnoće neizmirenja obaveza preduzeća iz nefinansijskog sektora u Srbiji i odgovarajuće premije za rizik. Na osnovu dobijenih rezultata izvršena je procena potencijala ovakvog proizvoda na lokalnom tržištu.

Ključne reči:

premija za rizik,
procena zasnovana na riziku,
obrnuti faktoring.